AS1056 - Mathematics for Actuarial Science. Chapter 2, Tutorial 2.

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Refreshing some concepts: manipulating inequalities

Rule 1. Adding/subtracting the same quantity from both sides of an inequality leaves the inequality symbol unchanged.
Rule 2. Multiplying/dividing both sides by a positive number leaves the inequality symbol unchanged.
Rule 3. Multiplying/dividing both sides by a negative number reverses the inequality.
Rule 4. Applying any *monotonically increasing/decreasing* function to an inequality leaves the inequality symbol unchanged / reverses the inequality.

$$\begin{cases} x \leq y \iff f(x) \leq f(y), & \text{if } f \text{ is increasing,} \\ x \leq y \iff f(x) \geq f(y), & \text{if } f \text{ is decreasing.} \end{cases}$$

In particular, raising both sides of an inequality to a power $n > 0$, when a and b are positive real numbers:

- $0 \leq a \leq b \iff 0 \leq a^n \leq b^n$
- $0 \leq a \leq b \iff a^{-n} \geq b^{-n} \geq 0.$

And raising both sides of an inequality to a power $n > 0$, when a and b are negative real numbers:

- $a \leq b \leq 0 \iff a^n \geq b^n \geq 0$
- $a \leq b \leq 0 \iff a^{-n} \leq b^{-n} \leq 0$

Absolute value / Modulus

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

1. Review

2. Exercises

Exercise 2.7.

(i) Give a graphical illustration of the solution space of the inequalities

$$y \geq 0$$
$$\frac{1}{4}x^2 + \frac{1}{9}y^2 > 1$$
$$6x - y \geq 0$$

(ii) Suggest one further linear inequality which, when added to the others, would result in a solution space which has a finite, non-zero area.

The equation of a circle of radius r and centre the origin is

$$\boxed{x^2 + y^2 = r^2}$$

which is a special case of the standard ellipse centred at the origin with width $2a$ and height $2b$:

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

Exercise 2.11.

Consider the simultaneous inequalities:

$$\begin{cases} x - |y| \geq 7 \\ y \geq A + x^2 \end{cases}$$

For which values of A is the solution space empty?
Hint: It might be helpful to draw a diagram.

Exercise 2.9.

Evaluate each of the following and give the limit as $K \to \infty$:

(iii) $\int_{-K}^{K} \lambda |x| e^{-\lambda x} dx$

L'Hôpital's rule

If:

1. $f(x)$, $g(x)$ are differentiable,
2. $\frac{d}{dx} g(x) \neq 0$, and,
3. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$

then,

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{\frac{d}{dx} f(x)}{\frac{d}{dx} g(x)} = L$$

Where c and L are any real number or $\pm\infty$.

➥ Though, often, limits are intuitive and don't require any advanced method!

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