AS1056 - Mathematics for Actuarial Science. Chapter 15, Tutorial 1.

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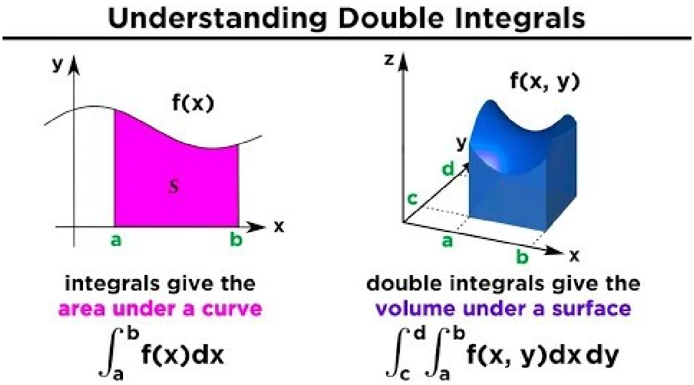
Refreshing some concepts:



Trigonometric identities

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ (double angle formulas for sine and cosine)
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$ (double angle formulas for sine and cosine)
- $\cos^2(\theta) + \sin^2(\theta) = 1$ (Pythagorean formula for sines and cosines)

Polar representation (see 13.5.2)

Any point (x, y) in two-dimensional space can be written in the form $(r\cos(\theta), r\sin(\theta))$, where $r = \sqrt{x^2 + y^2}$ and $\tan(\theta) = y/x$.

Jacobian

$dxdy = rdrd\theta$

Exercise 15.5

Use a transformation to polar coordinates to calculate the integral of $f(x, y) = \frac{(x-y)^2}{\sqrt{x^2+y^2}}$ over the unit disc, $\{(x, y) : x^2 + y^2 < 1\}$.

Exercise 15.9

Let A denote the area of the plane defined by $\frac{1}{K} < x < K$, $\frac{1}{K} < y < K$, $xy < 1$, where $K > 1$ is a constant.

(i) Sketch the area A.

(ii) Calculate $\iint_A (xy)^c dxdy$, where $c \neq -1$ is a constant.

(iii) Is there any value of c for which this integral converges to a limit as $K \to \infty$?

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