AS1056 - Mathematics for Actuarial Science. Chapter 13, Tutorial 1.

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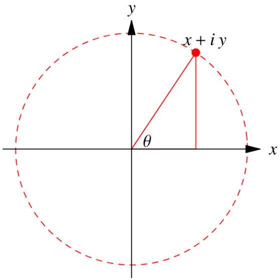
Refreshing some concepts: Argand Diagram

- An **Argand diagram** is a plot of complex numbers as points $z = x + iy$ in the complex plane using the x-axis as the real axis and y-axis as the imaginary axis.



The radius of the dashed circle represents the complex modulus $r = |z|$, and the angle $\theta = \arg(z)$ represents its complex argument.

That is,

- The complex number $z = x + iy$ is represented as the point (x, y) in the plane.
- $r = |z| = |x + iy| = \sqrt{x^2 + y^2}$ is the distance of the point (x, y) from the origin ($\sim$ equation of a circle centred at $(0, 0)$).

Polar representation

Any point (x, y) in two-dimensional space can be written in the form $(r\cos(\theta), r\sin(\theta))$, where:

- $r = |z| = \sqrt{x^2 + y^2}$
- $r\cos(\theta) = x$, $r\sin(\theta) = y$, $\tan(\theta) = \frac{y}{x}$

So we can write: $z = \underbrace{x + iy}_{\text{Cartesian representation}} = \underbrace{r \times (\cos(\theta) + i\sin(\theta)) \overset{\text{Euler's formula}}{=} re^{i\theta}}_{\text{Polar representation}}$.

with complex conjugate: $z^* = x - iy = r \times (\cos(\theta) - i\sin(\theta)) = re^{-i\theta}$.

$$\begin{cases} \textbf{Cartesian coordinates:} & (x, y) \\ \textbf{Polar coordinates:} & (r, \theta) \end{cases}$$

Exercise 13.7

Represent in polar form:

(i) $$\frac{(1+i)(2+i)}{3-i}$$

(ii) $$\sqrt{2+2i} - \sqrt{2-2i}$$

Exercise 13.13

Consider the polynomial

$$p(x) = x^4 + 8x^3 + 33x^2 + 68x + 52$$

Knowing that one of the roots of p is $x = 2 + 3i$, and all the other roots.
[**Hint:** Recall that complex solutions of polynomial equations always come in complex conjugate pairs.]
$\longrightarrow$ Quadratic polynomials with no real roots will have two complex roots that are conjugates of each other.

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