AS1056 - Mathematics for Actuarial Science. Chapter 0, Tutorial 1.

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Administrative Details

About myself:

- Welcome everyone! My name is Emilio Luis Sáenz Guillén, I'll be one of your Teaching Assistants for AS1056 - Mathematics for Actuarial Science.
- Doctoral Researcher in Actuarial Science at Bayes Business School.
- Email: emilio.saenz-guillen@bayes.city.ac.uk.
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Tutorials' details:

- Cover around 2-3 questions every week.
- Requirement: be *minimally* familiar with the corresponding week's topics and, if possible, also with the respective exercises.
- Solutions and tutorial notes will be uploaded to Moodle.

A. Set Theory

- **Symmetric difference**:

$$A \Delta B = (A \backslash B) \cup (B \backslash A) = (A \cap B^c) \cup (B \cap A^c)$$

- **De Morgan's laws**:

$$(A \cup B)^c = A^c \cap B^c \quad \text{and} \quad (A \cap B)^c = A^c \cup B^c$$

- **Intersection** is:
  - *Commutative*: $A \cap B = B \cap A$
  - *Associative*: $(A \cap B) \cap C = A \cap (B \cap C)$
  - *Distributive over union*: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
- **Union** is:
  - *Commutative*: $A \cup B = B \cup A$
  - *Associative*: $(A \cup B) \cup C = A \cup (B \cup C)$
  - *Distributive over intersection*: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Exercise A.6

A, B and C are three sets. Sets E and F are defined by

$$E = (A \cap B)\Delta C, \quad F = A \cap (B \Delta C).$$

Show that $F \subset E$. Find an expression for $E \backslash F$ in terms of A, B and C.

B.4. Rational functions

- A function f(x) is called a **rational function** if it has the form:

$$f(x) = \frac{p(x)}{q(x)}$$

  for $p(x)$ and $q(x)$ being polynomial functions.
- $p(x)/q(x)$ is a proper rational function if the degree of p is less than the degree of q.

- We can always reduce a rational function to a polynomial (quotient) plus a proper rational function (remainder). That is, take the improper rational function $p(x)/q(x)$, then we can re-express it as:

$$\frac{p(x)}{q(x)} = \underbrace{s(x)}_{\text{quotient}} + \frac{\overbrace{r(x)}^{\text{remainder}}}{q(x)}$$

where $\frac{r(x)}{q(x)}$ is now a proper rational function.

Partial Fractions Decomposition

We can express a *proper rational function*, $p(x)/q(x)$, as a sum of simpler fractions.

Form of the rational function	Form of the partial fraction
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$
*where $x^2 + bx + c$ cannot be factorised further	

Table 1: Partial fraction decomposition of various rational functions

Exercise B.4 Write the function

$$f(x) = \frac{4x^4}{(2x-1)^2(x+1)}$$

as a linear term in x plus a remainder expressed in partial fractions.

D.2. Combinations

$$^{n}C_{r} = \binom{n}{r} = \frac{^{n}P_{r}}{^{r}P_{r}} = \frac{n!}{r!(n-r)!}$$

❊ Number of possible arrangements in a collection of items where the order does not matter.

D.2.1. Pascal's triangle

$$
\begin{array}{ccccccccccccccc}
 & & & & & & & {}^{0}C_0 & & & & & & & \\
 & & & & & & {}^{1}C_0 & & {}^{1}C_1 & & & & & & \\
 & & & & & {}^{2}C_0 & & {}^{2}C_1 & & {}^{2}C_2 & & & & & \\
 & & & & {}^{3}C_0 & & {}^{3}C_1 & & {}^{3}C_2 & & {}^{3}C_3 & & & & \\
 & & & {}^{4}C_0 & & {}^{4}C_1 & & {}^{4}C_2 & & {}^{4}C_3 & & {}^{4}C_4 & & & \\
 & & {}^{5}C_0 & & {}^{5}C_1 & & {}^{5}C_2 & & {}^{5}C_3 & & {}^{5}C_4 & & {}^{5}C_5 & & \\
 & {}^{6}C_0 & & {}^{6}C_1 & & {}^{6}C_2 & & {}^{6}C_3 & & {}^{6}C_4 & & {}^{6}C_5 & & {}^{6}C_6 & \\
{}^{7}C_0 & & {}^{7}C_1 & & {}^{7}C_2 & & {}^{7}C_3 & & {}^{7}C_4 & & {}^{7}C_5 & & {}^{7}C_6 & & {}^{7}C_7 \\
 & & & & & & & \vdots & & & & & & &
\end{array}
=
\begin{array}{ccccccccccccccc}
 & & & & & & & 1 & & & & & & & \\
 & & & & & & 1 & & 1 & & & & & & \\
 & & & & & 1 & & 2 & & 1 & & & & & \\
 & & & & 1 & & 3 & & 3 & & 1 & & & & \\
 & & & 1 & & 4 & & 6 & & 4 & & 1 & & & \\
 & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \\
 & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
1 & & 7 & & 21 & & 35 & & 35 & & 21 & & 7 & & 1 \\
 & & & & & & & \vdots & & & & & & &
\end{array}
$$

Exercise D.2 The smallest integer which makes 3 appearances in Pascal's triangle is 6.

(i) Show that 6 makes exactly 3 appearances, i.e., that it cannot occur again lower down in the triangle.

(ii) Which is the smallest integer to make 4 appearances?

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