AS1056 - Mathematics for Actuarial Science. Chapter 7, Tutorial 2.

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Refreshing some concepts

Manipulating inequalities

Rule 1. Adding/subtracting the same quantity from both sides of an inequality leaves the inequality symbol unchanged.

Rule 2. Multiplying/dividing both sides by a positive number leaves the inequality symbol unchanged.

Rule 3. Multiplying/dividing both sides by a negative number reverses the inequality.

Rule 4. Squaring both sides of an inequality if both sides are positive/negative leaves the inequality symbol unchanged/ reverses the inequality.

Absolute value/Modulus

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

Exercise 7.3.

For $0 < h < 1$ calculate

$$\int_h^2 |x \ln(x)| dx$$

Does this converge to a finite limit as $h \to 0$?

Exercise 7.9.

(i) Give a graphical illustration of the solution space of the inequalities

$$y \geq 0$$
$$\frac{1}{4}x^2 + \frac{1}{9}y^2 > 1$$
$$6x - y \geq 0$$

(ii) Suggest one further linear inequality which, when added to the others, would result in a solution space which has a finite, non-zero area.

The equation of a circle of radius r and centre the origin is

$$x^2 + y^2 = r^2$$

which is a special case of the standard ellipse centred at the origin with width $2a$ and height $2b$:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Exercise 7.13.

Consider the simultaneous inequalities:

$$\begin{cases} x - |y| \geq 7 \\ y \geq A + x^2 \end{cases}$$

For which values of A is the solution space empty?
Hint: It might be helpful to draw a diagram.

Exercise 7.11.

Evaluate each of the following and give the limit as $K \to \infty$:

(iii) $\int_{-K}^{K} \lambda |x| e^{-\lambda x} dx$