AS1056 - Mathematics for Actuarial Science. Chapter 0, Tutorial 2.

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Administrative Details

About me:

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Course Details:

- Cover *at least* 5-6 questions every week (from week 2 to week 11)
- Requirement: be familiar with the exercises and we will solve them in class.
- Formal solutions will be uploaded into Moodle each Friday.

Exercise B.2

Suppose $\sqrt{2} = \frac{p}{q}$, where $\frac{p}{q}$ is a rational number in its lowest terms.
Tasks:

1. Show that p must be divisible by 2.

2. Show that q must be divisible by 2.

Question: What can you conclude from the above results?
Hint: Consider squaring both sides of the equation and analyze the resulting equation for possible contradictions.

Exercise B.4 (ii)

Use interval notation, along with ∪, ∩ and ^c if necessary, to write down the set of real numbers between 1 and 10 which have an even integer part and a non-zero fractional part.

Exercise B.5

Tasks:

1. Prove that the union of two countable sets is countable.

2. Consider $\mathbb{R} \setminus \mathbb{Q}$, i.e., the irrational numbers. If you take away the infinitely many rational numbers from the infinitely many real numbers, are you left with:

   (a) the empty set?
   (b) a countably infinite set?
   (c) an uncountably infinite set?

Hint: For part 1., think about how you can create a sequence that enumerates the elements of the union of the two sets. For part 2., consider the cardinality of real numbers and rational numbers and the nature of irrational numbers in comparison.

Exercise C.4

Write the function

$$f(x) = \frac{4x^4}{(2x-1)^2(x+1)}$$

as a linear term in x plus a remainder expressed in partial fractions.

Exercise D.3

The first 10 terms of an arithmetic sequence sum to 120. The next 10 terms sum to 400. Which is the first term in the sequence whose value is an integer?

Exercise D.5

The sum to infinity of a geometric series is 36. The first two terms sum to 27. What is the fourth term?

Exercise E.2

The smallest integer which makes 3 appearances in Pascal's triangle is 6.
(i) Show that 6 makes exactly 3 appearances, i.e., that it cannot occur again lower down in the triangle.